CHAPTER 7: REPRESENTATION OF TECHNICAL INFORMATION (CONT')

In this lesson we learn:

1. Linear Equations
2. Power Equations
3. Exponential Equations
4. Interpolation and extrapolation

1.0 INTRODUCTION - TRANSFORMING NONLINEAR EQUATIONS INTO LINEAR EQUATIONS

To an engineer, there is something marvelous about a straight line. One can always argue about a curve; does it fit this equation or that? But if the data plot on a straight line, there is no question; a straight line is a straight line. If the data do not plot on a straight line, then there is a problem. Therefore, engineers often manipulate nonlinear equations into a linear form. The following discussion presents the equations for linear equations, and those for power equations and exponential equations. Power and exponential equations are means by which non-linear data are made 'linear'.

2.0 LINEAR EQUATIONS

When experimental data plot as a straight line on rectangular grid paper, the equation of the line belongs to a family of curves whose basic equation is given by:

$$y = mx + b$$

where m is the slope of the line (a constant), and b is a constant referred to as the y *intercept* (the value of y when $x = 0$).

Figure 1
The straight line established by points (x_1, y_1) and (x_2, y_2).

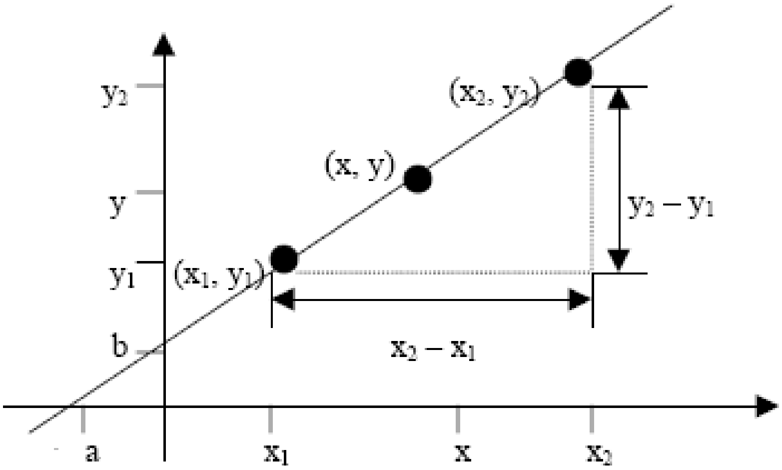

Figure 1 above shows that two distinct point (x_1, y_1) and (x_2, y_2) establish a straight line. The point (x, y) is an arbitrary point on the line.

The *slope*, m, of this line is defined as "rise over run", or

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (1)$$

Because all the quantities in this equation are known, the slope can be calculated. (Note: This equation in only valid for $x_2 \neq x_1$ to avoid division by zero. If $x_2 = x_1$, then the line is vertical, with the equation $x = x_1 = x_2$.)

The equation for the slope may also be written using the arbitrary point (x, y):

$$m = \frac{y - y_1}{x - x_1} \qquad (2)$$

Both sides of this equation may be multiplied by $(x - x_1)$, yielding

$$y - y_1 = m(x - x_1) = mx - mx_1 \qquad (3)$$

$$y = mx + y_1 - mx_1 \qquad (4)$$

If the constant b is defined as $(y_1 - mx_1)$, this equation becomes

$$y = mx + b \qquad (5)$$

The constant b is interpreted as the y-intercept because x= 0 when y= b. The x-intercept, a, is where y= 0. From equation 8-6, it is easy to show that

$$a = -\frac{b}{m} \qquad (6)$$

You can use these equations to determine the slope of the line, as well as the equation of the line plotted in the last lecture.

3.0 POWER EQUATIONS

When experimentally collected data are plotted on rectangular coordinate graph paper and the points do not form a straight line, you must next determine which family of curves the line most closely approximates.

A power equation has the form

$$y = kx^m \qquad (1)$$

If a logarithm is taken of both sides, then the power equation becomes linear:

$\log y = \log(kx^m) = \log(x^m k)$
$\log y = \log x^m + \log k$

$$\log y = m\log x + \log k \qquad (2)$$

Thus a plot of log y versus log x gives a straight line with a slope m and y-intercept log k, which is analogous to b in the equation $y = mx + b$. One can derive this linear equation using any desired base (2, *e* , or 10).

If the exponent *m* is positive, then the power equation plots as a *parabola*. The following table shows some selected values of x and y (for the equation $y = 2x^{0.5}$), along with the corresponding logarithms.

x	Y	log x	log y
1	2.000	0.000	0.301
2	2.828	0.301	0.452
3	3.464	0.477	0.540
5	4.472	0.699	0.651
10	6.325	1.000	0.801
25	10.000	1.398	1.000

When plotting these data, we have a choice. We may plot y versus x directly on a *log-log graph*, or we may plot log y versus log x on a *rectilinear graph*. The advantage of a log-log graph is that x and y may be read directly from the axes. Also, it eliminates the need to calculate the logarithms; in effect, the logarithmic axis is doing the calculation for you.

Unfortunately, the slope of the log-log graph is not meaningful. (See this for yourself. Determine the slope at two places on the line, and you will see that they differ). The advantage of the rectilinear graph is that the slope is meaningful. A rectilinear graph is particularly useful for plotting experimental data where the exponent *m* will be determined from the measured slope and the constant *k* will be determined from the measured y-intercept.

If the exponent *m* is negative, then the power equation plots as a *hyperbola*.

$$y = 10x^{-0.8} \qquad (3)$$

This equation becomes linear when logarithms are taken of both sides:

$$\log y = -0.8\log x + \log 10 \qquad (4)$$

The following table shows some selected values of x and y for this equation, and the corresponding logarithms.

x	y	log x	log y
1	10.000	0.000	1.000
2	5.743	0.301	0.759
3	4.152	0.477	0.618
4	3.299	0.602	0.518
5	2.759	0.699	0.441
10	1.585	1.000	0.200

4.0 EXPONENTIAL EQUATIONS.

Suppose your data do not plot as a straight line (or nearly straight) on rectangular coordinate paper nor is the line approximately straight on log-log paper. Without experience in analyzing experimental data, you may feel lost as to how to proceed. Normally, when experiments are conducted you have an idea as to how the parameters are related and you are merely trying to quantify the relationship. If you plot your data on semi-log graph paper and it produces a reasonably straight line, then it has the form of an exponential equation.

$$y = kB^{mx}$$

where B is the desired base (e.g., 2, e, or 10). Assuming base 10 is used, this equation becomes

$$y = k10^{mx}$$

Logarithms (base 10) are taken of both sides to give a linear equation :

$$\log y = \log (k10^{mx}) = \log (10^{mx}k)$$

$$\log y = \log 10^{mx} + \log k$$

$$\log y = mx + \log k$$

Thus a plot of log y versus x gives a straight line with slope m and intercept log k.

Example:

Determine the equation for the following data:

Velocity V, m/s	Fuel Consumption FC, mm^3/s
10	25.2
20	44.6
30	71.7
40	115
50	202
60	367
70	608

5.0 INTERPOLATION AND EXTRAPOLATION

Interpolation is extending between the data points and ***extrapolation*** is extending beyond the data points. The smooth curve drawn between data points is actually an interpolation, because there are no data between the points. Provided there are a sufficiently large number of closely spaced data points, interpolation is safe.

Extrapolation, on the other hand, can be quite risky, particularly if the extrapolation extends far beyond the data.

6.0 LINEAR REGRESSION

In mathematics, we are normally given a formula from which we calculate numbers. If we reverse this (i.e., determine the formula from the numbers), the process is called ***regression***, meaning "going backward". If the formula we seek is the equation of a straight line, then the process is called ***linear regression.*** You will learn more about linear regression later on in your education.